Solution of m – inputs and n-outputs control systems synthesis problem using the Lyapunov gradient-speed vector function

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***Abstract*—This paper describes one method of research and synthesis of control systems with m-inputs and n-outputs by the output of the object by the gradient-speed method of Lyapunov vector functions. The task of the synthesis of the regulator and the observer is considered as a system that can provide the specified (desired) transition characteristics of a closed system.**

***Keywords*— Control systems, closed control systems, Lyaputnov vector function, gradient-speed method, outcome control system**

I. INTRODUCTION

The modern control tasks are characterized by the ever-increasing complexity of the control objects, the requirements for stability and high quality in the context of multidimensionality of the system.

The prevailing problem revolves around the creation of control systems that account for the object's output, while in practice only the state vector $x(t)$ is available for measurement, not the object's output $y(t)$. In this case, the state variables of the object themselves are not used in the control law, but their estimates obtained be using the observing device [1,2,3,4]. This in itself requires a control system to be built for the object output in the form of modal control [1,5]. The modal control on the output of an object implies complex and ambiguous calculations on the characteristic polynomial [6,7] of a closed system with a controller and an observer and the transformation of the matrix of the object to a triangular or block-diagonal form. At the same time, the non-special matrix of canonical transformations is determined by the eigenvectors of the object's matrix, and the roots of the characteristic polynomial of a closed system by the combination of the roots of the characteristic polynomial of the system with a model controller and the state numbers eigenvalues [1, 3, 6].

II. MATERIAL AND METHODS

This paper proposes a new method of research of stability and the synthesis of control systems with m-inputs and n-outputs for the output of an object based on the gradient-speed method of Lyapunov vector functions [8,9,10,11]. The investigation of stability of a closed control system by measuring the output of the object and the solution of the problem of the synthesis of the controller (determining the elements of the gain matrix) and the observer (calculating the elements of the matrix of the observing device) are both based on the direct Lyapunov [12,13,14]. The proposed gradient-velocity method of Lyapunov vector functions in the study of the control system with m-inputs and n-outputs on the object output eliminates complex and ambiguous calculations and transformations and allows you to determine the region of choice of parameters of the controller and the observer, providing the desired (desired) transition characteristics closed system.

III. RESULTS

Let the control system at the exit of the object be presented in the form:

$$\dot{x}(t) = Ax(t) + BKx(t) + BK\varepsilon(t), x(t_0) = x_0 \qquad (1)$$

$$\dot{\varepsilon}(t) = A\varepsilon(t), -LC\varepsilon(t), \varepsilon(t_0) = \varepsilon_0 \qquad (2)$$

Where

$$A = \begin{Vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{Vmatrix},$$

$$B = \begin{Vmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{1m} \\ b_{21} & b_{22} & b_{23} & \dots & b_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & b_{n3} & \dots & b_{nm} \end{Vmatrix},$$

$$C = \begin{Vmatrix} c_{11} & c_{12} & c_{13} & \dots & c_{1n} \\ c_{21} & c_{22} & c_{23} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ c_{l1} & c_{l2} & c_{l3} & \dots & c_{1n} \end{Vmatrix},$$

$$x(t) = \begin{Vmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_n(t) \end{Vmatrix}, \quad \varepsilon(t) = \begin{Vmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \dots \\ \varepsilon_n(t) \end{Vmatrix},$$

$$K = \begin{Vmatrix} k_{11} & k_{12} & k_{13} & \dots & k_{1n} \\ k_{21} & k_{22} & k_{23} & \dots & k_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ k_{m1} & k_{m2} & k_{m3} & \dots & k_{mn} \end{Vmatrix},$$

$$L = \begin{Vmatrix} l_{11} & l_{12} & l_{13} & \dots & l_{1l} \\ l_{21} & l_{22} & l_{23} & \dots & l_{2l} \\ \dots & \dots & \dots & \dots & \dots \\ l_{n1} & l_{n2} & l_{n3} & \dots & l_{nl} \end{Vmatrix}$$

Us (1) and (2) can be expanded as

$$\begin{cases} \dot{x}_1 = (-a_{i1} - \sum_{j=1}^{m} b_{ij}k_{j1})x_1 + (-a_{12} - \sum_{j=1}^{m} b_{ij}k_{j2})x_2 + \\ (-a_{13} - \sum_{j=1}^{m} b_{ij}k_{j3})x_3 + \dots + \\ + (-a_{1n} - \sum_{j=1}^{m} b_{ij}k_{jn})x_n + (\sum_{j=1}^{m} b_{ij}k_{j1})\varepsilon_1 + (\sum_{j=1}^{m} b_{ij}k_{j2})\varepsilon_2 + \\ (\sum_{j=1}^{m} b_{ij}k_{ij})\varepsilon_3 + \dots + (\sum_{j=1}^{m} b_{ij}k_{jn})\varepsilon_n, i = 1,\dots,n; \\ \dot{\varepsilon}_1 = (-a_{i1} - \sum_{j=1}^{l} l_{ij}c_{j1})\varepsilon_1 + (-a_{12} - \sum_{j=1}^{l} l_{ij}c_{j2})\varepsilon_2 + \\ (-a_{13} - \sum_{j=1}^{l} l_{ij}c_{j3})\varepsilon_3 + \dots + (-a_{1n} - \sum_{i=1}^{l} l_{ij}c_{jn})\varepsilon_n, i = 1,\dots,n \end{cases} \quad (3)$$

We find the condition of robust asiptotic stability of the system (3) by the gradient-velocity method of Lyapunov vector functions [8,9].

From (3) we find the components of the gradient vector for Lyapunov vector functions.

$$\begin{cases} \frac{\partial V_i(x,\varepsilon)}{\partial \varepsilon_k} = -(-a_{ik} - \sum_{j=1}^{m} b_{ij}k_{jk})x_k, i = 1,\dots,n; k = 1,\dots,n \\ \frac{\partial V_i(x,\varepsilon)}{\partial \varepsilon_k} = -(\sum_{j=1}^{m} b_{ij}k_{jk})\varepsilon_k, i = 1,\dots,n; k = 1,\dots,n \\ \frac{\partial V_{n+1}(x,\varepsilon)}{\partial \varepsilon_k} = -(-a_{ik} - \sum_{j=1}^{l} l_{ij}c_{jk})\varepsilon_k, i = 1,\dots,n; k = 1,\dots,n \end{cases} \quad (4)$$

(4) we decompose the velocity vector to coordinates $(x_1,\dots,x_n,\varepsilon_1,\dots\varepsilon_n)$

$$\begin{cases} (\frac{dx_i}{dt})x_k = (-a_{ik} - \sum_{j=1}^{m} b_{ij}k_{jk})x_k, i = 1,\dots,n; k = 1,\dots,n \\ (\frac{dx_i}{dt})\varepsilon_k = (\sum_{j=1}^{m} b_{ij}k_{jk})\varepsilon_k, i = 1,\dots,n; k = 1,\dots,n \\ (\frac{d\varepsilon_i}{dt})\varepsilon_k = (-a_{ik} - \sum_{j=1}^{l} l_{ij}c_{jk})\varepsilon_k, i = 1,\dots,n; k = 1,\dots,n \end{cases} \quad (5)$$

The total time derivative of Lyapunov functions is defined as the scalar product of the gradient vector (4) and the velocity vector (5)

$$\begin{aligned} \frac{dV(x,\varepsilon)}{dt} &= \sum_{i=1}^{n}\sum_{k=1}^{n} \frac{\partial V_i(x,\varepsilon)}{\partial x_k}\left(\frac{dx_i}{dt}\right)_{x_k} + \sum_{i=1}^{n}\sum_{k=1}^{n} \frac{\partial V_i(x,\varepsilon)}{\partial x_k}\left(\frac{dx_i}{dt}\right)_{\varepsilon_k} \\ &+ \sum_{i=1}^{n}\sum_{k=1}^{n} \frac{\partial V_{+i}(x,\varepsilon)}{\partial x_k}\left(\frac{d\varepsilon_i}{dt}\right)_{\varepsilon_k} = \\ &- \sum_{i=1}^{n}\sum_{k=1}^{n}(-a_{ik} - \sum_{j=1}^{m} b_{ij}k_{jk})^2 x_k^2 - \sum_{i=1}^{n}\sum_{k=1}^{n}(\sum_{j=1}^{m} b_{ij}k_{jk})^2 \varepsilon_k^2 - \\ & \sum_{i=1}^{n}\sum_{k=1}^{n}(-a_{ik} - \sum_{j=1}^{l} l_{ij}c_{jk})^2 \varepsilon_k^2, \end{aligned} \quad (6)$$

From (6) it follows that the total time derivative of Lyapunov vector functions is a sign-negative function. From (4), the Lyapunov function can be represented as:

$$\begin{aligned} V(x,\varepsilon) &= \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}(\sum_{j=1}^{m} b_{ij}k_{jk} + a_{ik})x_k^2 - \\ &- \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}[(\sum_{j=1}^{l} l_{ij}c_{jk} + a_{ik}) - \sum_{i=1}^{m} b_{ij}k_{jk})]\varepsilon_k^2, \end{aligned} \quad (7)$$

The condition for the existence of a Lyapunov function is defined by the inequalities:

$$\sum_{j=1}^{n} b_{ij}k_{jk} + a_{ik} > 0, i = 1,\dots,n; k = 1,\dots,n \quad (8)$$

$$\sum_{j=1}^{n} l_{ij}c_{jk} + a_{ik} - \sum_{j=1}^{m} b_{ij}k_{jk} + a_{ik} > 0, i = 1,\dots,n; k = 1,\dots,n \quad (9)$$

The system of inequalities (8) and (9) is a condition for the robust stability of the dynamic compensator. Condition (8) allows for robust stability of the system when controlled by a state vector. Suppose we have a closed control system of desired transients with m-inputs and n-outputs with matrix:

$$G = \begin{Vmatrix} -d_{11} & -d_{12} & -d_{13} & \dots & -d_{1n} \\ -d_{21} & -d_{22} & -d_{23} & \dots & -d_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ -d_{n1} & -d_{n2} & -d_{n3} & \dots & -d_{nn} \end{Vmatrix}$$

The task is to determine the coefficient of the regulator (elements of the matrix K) such that the elements of the matrix of a closed system have the specified values.

We study the stability of a system with given values of the coefficients by the gradient-speed method of Lyapunov vector functions.

For a system with given parameters, write the equations of state in expanded form:

$$\begin{cases} \dot{x}_1 = -d_{11}x_1 - d_{12}x_2 - d_{13}x_3 -,\dots,-d_{1n}x_n \\ \dot{x}_2 = -d_{21}x_1 - d_{22}x_2 - d_{23}x_3 -,\dots,-d_{2n}x_n \\ \dots \\ \dot{x}_n = -d_{n1}x_1 - d_{n2}x_2 - d_{n3}x_3 -,\dots,-d_{nn}x_n \end{cases} \quad (10)$$

From (10) we find the components of the gradient vector of Lyapunov vector functions

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_1} = d_{11}x_1, \frac{\partial V_1(x)}{\partial x_2} = d_{12}x_2, \frac{\partial V_1(x)}{\partial x_3} = d_{13}x_3,\dots, \frac{\partial V_1(x)}{\partial x_n} = d_{1n}x_n \\ \frac{\partial V_2(x)}{\partial x_1} = d_{21}x_1, \frac{\partial V_2(x)}{\partial x_2} = d_{22}x_2, \frac{\partial V_2(x)}{\partial x_3} = d_{23}x_3,\dots, \frac{\partial V_2(x)}{\partial x_n} = d_{2n}x_n \\ \dots \\ \frac{\partial V_n(x)}{\partial x_1} = d_{n1}x_1, \frac{\partial V_n(x)}{\partial x_2} = d_{n2}x_2, \frac{\partial V_n(x)}{\partial x_3} = d_{n3}x_3,\dots, \frac{\partial V_n(x)}{\partial x_n} = d_{nn}x_n \end{cases} \quad (11)$$

(3) we decompose the velocity vector to coordinates:

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_1} = d_{11}x_1, \frac{\partial V_1(x)}{\partial x_2} = d_{12}x_2, \frac{\partial V_1(x)}{\partial x_3} = d_{13}x_3,\dots, \frac{\partial V_1(x)}{\partial x_n} = d_{1n}x_n \\ \frac{\partial V_2(x)}{\partial x_1} = d_{21}x_1, \frac{\partial V_2(x)}{\partial x_2} = d_{22}x_2, \frac{\partial V_2(x)}{\partial x_3} = d_{23}x_3,\dots, \frac{\partial V_2(x)}{\partial x_n} = d_{2n}x_n \\ \dots \\ \frac{\partial V_n(x)}{\partial x_1} = d_{n1}x_1, \frac{\partial V_n(x)}{\partial x_2} = d_{n2}x_2, \frac{\partial V_n(x)}{\partial x_3} = d_{n3}x_3,\dots, \frac{\partial V_n(x)}{\partial x_n} = d_{nn}x_n \end{cases} \quad (12)$$

The total derivative of the Lyapunov vector-function with respect to time is determined by the expression:

$$\frac{\partial V(x)}{\partial t} = -d_{11}^2 x_1^2 - d_{12}^2 x_2^2 - d_{13}^2 x_3^2 -,...,-d_{1n}^2 x_n^2 - \\ - d_{21}^2 x_1^2 - d_{22}^2 x_2^2 - d_{23}^2 x_3^2 -,...,-d_{2n}^2 x_n^2 -,...,- \\ - d_{n1}^2 x_1^2 - d_{n2}^2 x_2^2 - d_{n3}^2 x_3^2 -,...,-d_{nn}^2 x_n^2 \quad (13)$$

From (13) it follows that the total time derivative of Lyapunov vector functions is a sign-negative function.

From (11), the Lyapunov function can be represented in scalar form:

$$V(x) = \frac{1}{2}(d_{11} + d_{21} +,...,+ d_{n1}) x_1^2 + \\ + \frac{1}{2}(d_{12} + d_{22} +,...,+ d_{n2}) x_2^2 +,...,+ \\ + \frac{1}{2}(d_{1n} + d_{2n} +,...,+ d_{nn}) x_n^2 \quad (14)$$

The condition for the existence of Lyapunov vector functions for system (10) is described by the system of inequalities:

$$\begin{cases} d_{11} + d_{21} +,...,+ d_{n1} > 0 \\ d_{12} + d_{22} +,...,+ d_{n2} > 0 \\ d_{13} + d_{23} +,...,+ d_{n3} > 0 \\ ... \\ d_{1n} + d_{2n} +,...,+ d_{nn} > 0 \end{cases} \quad (15)$$

We can equate inequalities (8) and (15) for system (1) and (2) to have the specified properties and from there find the required values of the coefficients of the matrix k:

$$\sum_{j=1}^{n} b_{ij} k_{jk} + a_{ik} = d_{ik}, i = 1,..., n; k = 1,..., n \quad (16)$$

$$\sum_{j=1}^{n} b_{ij} k_{jk} + a_{ik} - d_{ik} = 0, i = 1,..., n; k = 1,..., n \quad (17)$$

From this n-system of algebraic equations (17) we can find the values of the elements of the matrix k $(k_{ij}; i = 1,..., n; j = 1,..., n)$

System (9) can be rewritten in the form:

$$\sum_{j=1}^{n} l_{ij} c_{jk} + 2a_{ik} - d_{ik} = 0, i = 1,..., n; k = 1,..., n \quad (18)$$

Solving the system (18) with respect to the coefficients $l_{ij}; j = 1,..., n; i = 1,..., l)$, we can find the boundary values for the coefficients of the observing device.

Suppose, for simplicity, we assume that the matrices B, L and C have dimension, respectively $n \times 1, l \times 1, l \times 1$, and the matrices A and D are given a controlled canonical form. Then the equations can be rewritten in the form:

$$\sum_{j=1}^{n} k_j - \frac{1}{b_i}(-a_i + d_i) = 0, i = 1,..., n; \quad (19)$$

From (19), the boundary values of the coefficients $(l_i; i = 1,..., l)$ of the observability matrix will be determined:

$$l_i = \frac{2a_i + d_i}{\sum_{j=1}^{n}}, i = 1,..., n \quad (20)$$

Thus, the problem of synthesizing control systems with m-inputs and n-outputs by the object output is solved by the gradient-speed method of Lyapunov vector functions completely. The calculation of the coefficients of the controller and the observing device according to the desired characteristic of the system (10) is presented as a solution of the system of algebraic equations (17) and (18).

IV. Conclusions

The known methods of synthesis of the closed-loop control systems for the object's output are based on the model control of the object's output. The calculation of the matrix elements of the regulator and the observer requires certain regulated transformations - complex and ambiguous calculations of the roots of the characteristic equation of a closed system. The roots of the characteristic polynomial of a closed system are obtained by combining the roots of the system with a model controller and the eigenvalues of the state observer.

The gradient-speed method of Lyapunov vector functions allows one to solve the problem of m-input and n-output control systems directly using the elements of the matrix of the controller object and the observer.

The approach is proposed to determine the range of changes in the object parameters, a regulator, and an observer that provides the specified (desired) transient characteristics of a closed system.

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